Grade 5 2-digit x 2-digit multiplication

5.N.5	
Demonstrate an understanding of multiplication (2-digit numerals by 2-digit numerals) to solve problems.	1. Illustrate partial products in expanded notation for both factors (e.g., for 36 x 42, determine the partial products for [30 + 6] x [40 + 2]). 2. Represent both 2-digit factors in expanded notation to illustrate the distributive property (e.g., to determine the partial products of 36 x 42, [30 + 6] x [40 + 2] = 30 x 40 + 30 x 2 + 6 x 40 + 6 x 2 = 1200 + 60 + 240 + 12 = 1512). 3. Model the steps for multiplying 2-digit factors using an array and base-10 blocks, and record the process symbolically. 4. Describe a solution procedure for determining the product of two 2-digit factors using a pictorial representation, such as an area model. 5. Solve a multiplication problem in context using personal strategies, and record the process.

Clarification of the outcome:

- The outcome is multiplying 2-digit x 2-digit whole numbers and solving problems involving 2-digit multiplication. It involves being able to do 2-digit multiplication (a skill) and understanding why the multiplication algorithm works (a concept matter).
- The more important matters, in terms of immediate need and future pay-off, are understanding the distributive principle and having proficiency in multiplying multiples of 10 and 100. These are grade 4 matters. If they are not in place, then DO NOT proceed with the grade 5 outcome until they are in place. Refer to Gr 4 Multiplication algorithm for assistance.
- Future pay-off? Algebraic thinking requires distributive thinking, for example in situations such as 2(x + 3) = 14 and (2a + 3b + 5)(a + 5) is the same as what expression.

Required close-to-at-hand prior knowledge:

- Automaticity of multiplication facts.
- Proficiency in multiplying by multiples of 10s and 100s.
- Understands the distributive principle in a functional way.

SET SCENE stage

The problem task to present to students:

Present students with a problem involving 2-digit x 3-digit multiplication. Here is an example.

Johnny is 12 years old today. If you ignore leap years, how many days old is Johnny?

Ask students to solve the problem any way they want (using place value materials, etc.), but not by using a calculator.

Comments

Students will likely not be able to do the 2-d x 3-d multiplication. That is okay. The important thing is that they realize they need to multiply to obtain the solution and that they attempt to invent a method for doing the multiplication.

The problem goes beyond the expectations of the outcome. However, it is important to do so. Life is not restricted to 2-digit x 2-digit multiplication situations. Going beyond the outcome provides an opportunity for students to realize that what they have learned can be extended to any number of digits, in this case, 2-d x 3-d multiplication. This opportunity arises in the DEVELOP lesson activity that revisits SET SCENE.

DEVELOP stage

Comments

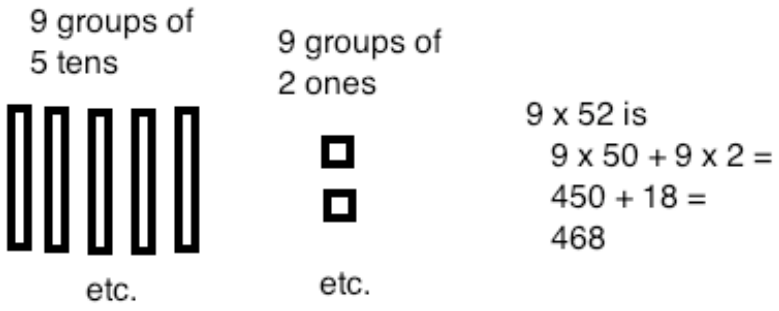
The formula for the area of a rectangle (base x height) is a grade 6 outcome. This means that you cannot develop the grade 5 whole number multiplication algorithm by thinking in terms of the area of a rectangle. You can think in terms of the number of dots in an array. Thus, when a rectangle is drawn for a multiplication question (e.g. 15 x 12), the understanding is that it refers to an array of 15 dots by 12 dots. We just don't bother drawing the dots.

Activity 1: Revisits SET SCENE and addresses indicator 5.

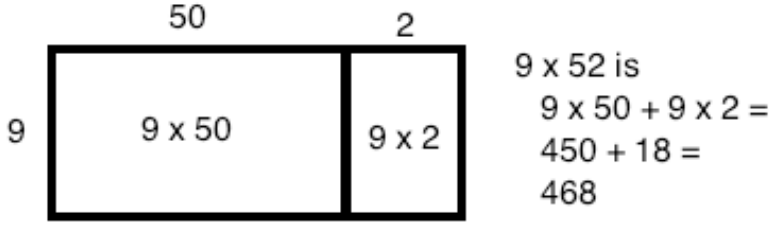
- Revisit the SET SCENE task by asking selected students to present their thoughts and efforts at solving the problem. Accept what they tell you.
- ENSURE they realize multiplication is the wisest way to view the problem. Translate the problem into whole number multiplication (12 x 365). Discuss situations where it might be necessary to multiply 2-digit whole numbers by 2-digit or more whole numbers. Discuss that being able to do "big" multiplications without a calculator lets you be boss over a calculator.

Activity 2: Addresses achievement indicators 1 through 5, and revisits grade 4 outcome.

- Present a problem that involves 1-digit by 2-digit multiplication. Here is an example: "*Mary is in grade 4. She is 9 years old. How many weeks old is she?*"
- Ask students to represent the problem with a number sentence. Expect '9 x 52 = ?'. Ask students to use PV materials to represent the multiplication and to show the work, using numbers. Assist as required. Discuss results. [NOTE: PV materials force students to think along place value lines. Refer to the diagram.]

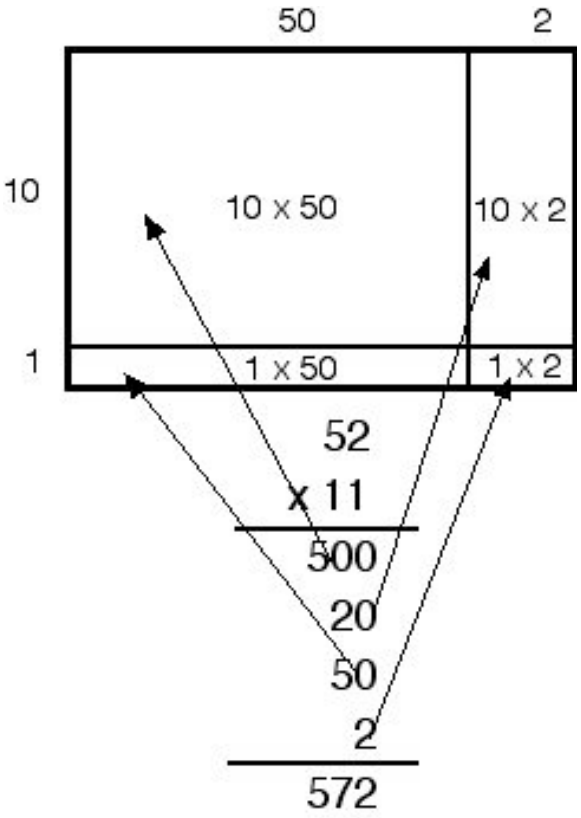


- Ask students to represent 9 x 52 with the array of dots (shown by a rectangle). Ask them to split 52 along place value lines. Ask them to figure out the answer. Assist as required. Discuss results. [Refer to the diagram.]



Activity 3: Addresses achievement indicators 1 through 5.

- Assuming that most students in grade 5 are 11 years old, present a problem that involves 2-digit by 2-digit multiplication such as: "*How many weeks old are you, if you are 11 years old?*"
- Ask them to represent the problem with a number sentence. Expect '11 x 52 = ?'. Ask them to use a rectangle to represent the array for 11 x 52. Ask them to split the rectangle along place value lines for the number of rows (11) and the number of columns (52). Ask students to show the work by writing numbers vertically. Assist as required. [Refer to the diagram.]



- Ask students to show the work by writing numbers horizontally. Assist as required. Expect:

  11 x 52

  = (10 + 1) x (50 + 2)

  = 10 x 50 + 10 x 2 + 1 x 50 + 1 x 2

  = 500 + 20 + 50 + 2.

  Ensure they realize where each part comes from in relation to the rectangle model.

- Repeat for two other 2-digit x 2-digit multiplication questions.

Comments

Students should be comfortable with both the vertical and horizontal writing styles for showing the symbolic work. Each style has an advantage. The horizontal style mimics algebraic writing style and thus helps "prepare the garden" for students who may want to take pre-calculus later on.

The vertical style can make the addition easier for some students because it allows for better visual organization of place value positions.

Activity 4: Addresses achievement indicators 1, 2, and 5, and practice.

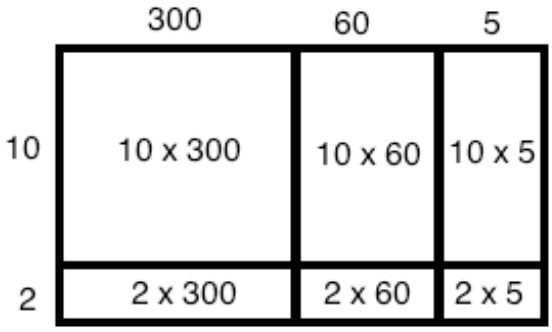
- Organize students into groups. Have each group make up two 2-digit x 2-digit problems (actual problems, not just multiplication questions). For example, a group might make up this problem: "*The teacher has 27 boxes of pencils in her desk. Each box has 35 pencils. How many pencils does the teacher have in her desk?*"
- Have each group solve the problems, using the vertical writing style for one problem and the horizontal style for the other problem. Encourage students not to use a rectangle model to show the multiplication. Rather, encourage them to think about splitting along place value lines by making mental images (e.g. 78 is 70 + 8) of the numbers.
- Have the groups exchange their problems for other groups to solve. Have selected groups present their problems and solutions.

Comments

It is time to start weaning students away from models and move them towards thinking about and working with numbers in a symbolic way. However, some students may not be ready for that step. Do not force them to abandon models if they are not ready to do so yet.

Activity 5: Revisits SET SCENE and addresses indicators 1, 2, 4, and 5, and practice.

- Revisit the SET SCENE task and ask students to draw a rectangle model showing the multiplication (e.g. 12 x 365). Assist as required.
- Ask students to use the rectangle model to show the multiplication with the numbers split along place value lines. Have them obtain the answer symbolically, using the horizontal and vertical writing styles. Assist as required. Discuss results. [Refer to diagram.]



3000
600
50
600
120
10
4380

(10 + 2) x (300 + 60 + 5)
3000 + 600 + 50 + 600 + 120 + 10
4380

- Ask students to create another 2-digit x 3-digit problem. Agree upon a problem and use it for the whole class. Ask students to: (1) represent the problem with a multiplication statement, (2) draw a rectangle model showing the multiplication, (3) split the numbers along place value lines, and (4) obtain the answer symbolically, using the horizontal and vertical writing styles. Discuss results.
- Discuss why understanding how to do 2-d x 2-d multiplication (and now 2-d x 3-d multiplication) gives them the power to multiply really big numbers.

Activity 6: Assessment of teaching.

- Present two 2-digit by 2-digit multiplication questions. Ask students to do the arithmetic by using the vertical writing style for one question and the horizontal writing style for the other question.
- Have students select one of the questions. Have them explain the work by drawing a rectangle model for it.

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partial well-designed worksheet follows.

The worksheet contains a sampling of question types. More questions of each type are needed.

The MAINTAIN stage follows the sample worksheet.

Question 1.

For each multiplication, draw a rectangle diagram and use the horizontal writing style to figure out the answer.

a) 6 x 38

b) 5 x 94

Question 2.

For each multiplication, draw a rectangle diagram and use the vertical writing style to figure out the answer.

a) 4 x 57

b) 8 x 39

Question 3.

For each multiplication, draw a rectangle diagram and use the vertical writing style to figure out the answer.

a) 23 x 62

b) 18 x 75

Question 4.

For each multiplication, draw a rectangle diagram and use the horizontal writing style to figure out the answer.

a) 17 x 38

b) 42 x 73

Question 5.

Solve the problem. Do the work symbolically, using either the vertical or the horizontal writing style.

Johnny has 38 bags with 45 candies in each. How many candies does Johnny have?

MAINTAIN stage

Mini-task example

Every so often:

- Provide a 2-digit x 2-digit multiplication problem (a problem, not merely a multiplication question). Ask students to solve the problem.

Rich-task example

Have students calculate the answer to 11 x 32, to 11 x 54, and to 11 x 26, without using a calculator. Do you see a pattern? Can you predict what the answer to 11 x 35 will be? Confirm your prediction by using a calculator to calculate 11 x 35.

Comments

This is a rich-task because it integrates 2-digit multiplication with patterning.